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Resonant excitation of ion plasma waves by VLF and whistler waves

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Abstract. The reactive mechanism of nonlinear wave mixing has been used to excite ion plasma oscillations in a nonisothermal magnetoplasma. Input waves are transverse and belong to the low frequency regime. It is shown that the resonance conditions are relatively easier to satisfy over a wide range of plasma and wave parameters in the case of the mixing of whistlers. The amplitude of the generated signal is appreciably high at moderately strong LF waves. With increasing generated frequency it shows a continuous decrease.

1. Introduction

The excitation of longitudinal oscillations due to the interaction of charged particle beams and electromagnetic waves with plasma has drawn considerable attention in recent years (Akhiezer *et al* 1967, Chen *et al* 1965, Arunasalam and Brown 1964, Drummond 1962, Ichimaru *et al* 1962). These oscillations play an important role in the phenomenon of the scattering of electromagnetic waves, which is conversely used to study the natural modes of plasma oscillations. Kroll *et al* (1964) have investigated the phenomenon of the excitation of plasma waves due to two externally applied high frequency EM waves, and have shown that under the resonant conditions, the density fluctuations are very high in amplitude. Kuhn *et al* (1968) were able to generate space charge waves at ultrahigh frequencies by the resonant nonlinear interaction of two microwave signals in a bounded magnetoplasma. However, since the resonant conditions are not easily satisfied in isothermal plasmas, the work reported thus far has been mainly limited to high frequencies.

This paper outlines a treatment of the nonlinear interaction of very low frequency EM waves with a plasma, to excite the natural modes of oscillations. The plasma is taken to be collisionless, nonisothermal and is considered to be immersed in a strong magnetic field. The case of a nonisothermal plasma has been chosen to avoid the ion–Landau damping of the excited mode; the condition for weak damping is that the ion thermal velocity must be much less than the phase velocity of the disturbance and is satisfied when the electron temperature is much higher than that of ions (Stix 1962). A strong magnetic field is applied in the direction of wave propagation to make the penetration of low frequency EM waves possible in the plasma. In this geometry of wave propagation, the propagating modes of EM waves are circularly polarized in the transverse plane. The source of nonlinearity in our analysis is the mutual interaction of carrier current density with the magnetic field of the EM wave. Since the interaction of a current of constant magnitude with a magnetic field of constant magnitude when the phase difference between them is constant does not give rise to time varying currents or fields, the phenomenon of the excitation of oscillations involves either two modes of a low frequency EM wave or the two waves of different frequencies. These processes have been discussed in the following sections and it is shown that in the case of the two mode interaction of a VLF wave the resonance conditions are satisfied when the plasma is dense. The amplitude of the resulting oscillation is appreciably high. In the case of two wave mixing (mixing of oppositely propagating whistlers) the resonance conditions are seen to be satisfied over a wide range of plasma and wave parameters.

2. Dispersion relation and conditions for resonance

On neglecting the collisional and Landau damping the dispersion relations for low frequency oscillations in a nonisothermal plasma (for $\boldsymbol{k}$ parallel to a static magnetic field $\boldsymbol{B}_0$) can be written as (Stix 1962)

$$\frac{\omega^2}{k^2} = c^2\left(1-\frac{\omega_p^2}{(\omega\mp\omega_{ce})(\omega\pm\omega_{ci})}\right)^{-1} \tag{1}$$

for EM waves, $\boldsymbol{E}\perp\boldsymbol{B}_0$ and

$$\frac{\omega^2}{k^2} = \left(1-\frac{\omega^2}{\omega_{pi}^2}\right)\frac{2KT_e}{m}\frac{m}{m_i} \tag{2}$$

for longitudinal plasma oscillations when $\boldsymbol{E}\parallel\boldsymbol{B}_0$ and $T_e \gg T_i$, where $\omega_p = (4\pi Ne^2/m)^{1/2}$ is the electron plasma frequency, $-e$ the electronic charge, $\omega_{ce} = |e|B_0/mc$ the electron cyclotron frequency, T_e the temperature of electrons and the quantities with suffix i refer to ions. The two signs in equation (1) refer to the extraordinary and ordinary modes respectively and the corresponding wavevectors may be designated as k_e and k_o. At frequencies higher than the ion cyclotron frequency only the extraordinary mode propagates while below this both modes propagate. Equation (2) shows that the phase velocity of ion oscillations is very small; for a given ω, k is large. Therefore, the input waves must have either a very high carrier concentration or, if the mixing frequencies are high (lying between ion and electron cyclotron frequencies), then their directions must be opposite to each other so that the wavevector of the generated difference frequency wave is the sum of wavevectors of fundamental waves ($\omega' = \omega_1-\omega_2$ and $k' = k_e^1+k_e^2$) and the resonance condition may be satisfied. Based on these two possibilities the amplitude of the excited ion oscillation has been calculated in the following sections.

3. Mixing of two modes of a VLF wave in a dense plasma ($\omega < \omega_{ci}$)

The electric vectors of extraordinary and ordinary modes of a VLF wave may be written as (Ginzburg 1960)

$$A_1^1 = A_{10}^1 \exp\{\mathrm{i}(\omega t-k_e z)\}$$

and

$$A_2^1 = A_{20}^1 \exp\{\mathrm{i}(\omega t-k_o z)\}$$

where

$$A^1_{1,2} = E^1_x \pm iE^1_y.$$

The behaviour of the ion–plasma wave generated as a result of nonlinear mixing of these modes may be investigated by solving the Boltzmann equation for the carrier distribution function and Maxwell's equations for the electric vector. Following the analysis of Shkarofsky *et al* (1966) and Shkarofsky (1968) the expression for the electric vector of the generated wave of frequency 2ω becomes

$$E_z^{11} = \epsilon_l^{-1}\Phi A_1^1 A_2^1 \tag{3}$$

$$\begin{aligned}\Phi = &-\frac{5i}{9}(k_e+k_o)^{-2}\frac{e\omega_p^2}{m}\langle v^{-2}\rangle\left\{\frac{1}{4\omega}\left(\frac{k_e}{\omega+\omega_{ce}}+\frac{k_o}{\omega-\omega_{ce}}\right)\right.\\ &\left.-\frac{k_e+k_o}{10(\omega^2-\omega_{ce}^2)}-\frac{1}{5}\left(\frac{k_e}{(\omega-\omega_{ce})^2}+\frac{k_o}{(\omega+\omega_{ce})^2}\right)\right\}\\ &-\frac{ie\omega_p^2 k_e k_o}{6m\omega^2\omega_{ce}^2}(k_e+k_o)^{-1}-\frac{iem\omega_p^2}{32m_i^2\omega^3}\left(\frac{k_e}{\omega-\omega_{ci}}+\frac{k_o}{\omega+\omega_{ci}}\right)\end{aligned} \tag{4}$$

$$\epsilon_l = 1-\frac{\omega_p^2 m}{4\omega^2 m_i}+\frac{5}{9}\frac{\omega_p^2\langle v^{-2}\rangle}{(k_e+k_o)^2} \tag{5}$$

$$\langle v^{-2}\rangle = 4\pi\int_0^\infty f_0^0\,dv$$

and f_0^0 is the unperturbed isotropic part of the electron distribution function.

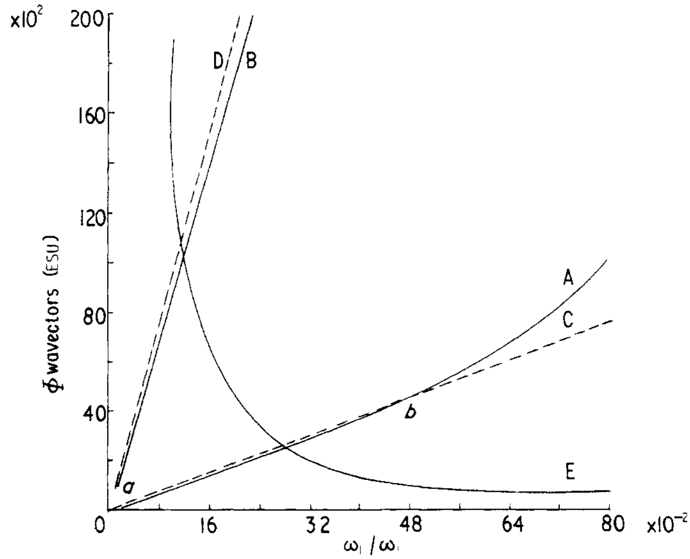

Figure 1. Wavevectors and Φ against the normalized frequency of the generated signal. Curves A and B refer to k_e+k_o (cf equation (1)) in units of ω_i/c, for $\omega_p/\omega_e = 10^2$ and 10^3. curves C and D refer to k' (cf equation (2)) in units of ω_i/c for $c\langle v^{-2}\rangle^{1/2} = 2\times10^2$ and 2×10^3. k' is the propagation vector of natural mode of the ion oscillation corresponding to the frequency on the abscissa. Curve E refers to Φ in units of $e\omega_p^3/10m\omega_e^2\omega_i^2c$; $\omega_{i,e} \equiv \omega_{ci,ce}$.

Equations (3) to (5) have been derived in the limit of the drift velocity of ions being much less than the phase velocity of the longitudinal disturbance and $|E_z^{1\,1}| < |A_1^1|, |A_2^1|$. It is necessary to point out here that at the singularity given by $\epsilon_l = 0$, our analysis is not valid. However, in the vicinity of this singularity, where our analysis is valid the amplitude of the excited ion oscillation can be seen to be high. The plot of the amplitude function Φ against fundamental wave frequency is given in figure 1. The electron temperature ($\simeq 10^4$ K) has been chosen such that $\epsilon_l^{-1} \simeq 0{\cdot}1$. The figure shows a continuous fall of Φ with ω; at higher values of ω saturation is seen. The magnitude of Φ (given in ESU) is appreciably high, therefore, for moderately strong fundamental waves, a yield of 10% is expected.

To have an idea of the accessibility of resonance conditions the $(k_e + k_o)$ against frequency curves and the dispersion curves for the natural mode of ion oscillation are plotted in figure 1. The points *a* and *b* where the two curves intersect each other give the parameters for resonance. Unfortunately nothing can be said with certainty in this region as the above analysis fails here.

4. Mixing of two oppositely propagating whistlers ($\omega_{ci} < \omega_1, \omega_2 < \omega_{ce}$)

The electric vectors of two oppositely propagating whistlers of frequencies ω_1 and ω_2 are written as

$$A_1^1 = A_{10}^1 \exp\{i(\omega_1 t - k_e^1 z)\}$$

and

$$A_1^2 = A_{10}^2 \exp\{i(\omega_2 t + k_e^2 z)\}$$

where $A_1^2 = E_x^2 + iE_y^2$; the superscript 2 refers to the frequency ω_2. The nonlinear interaction of these two waves gives rise to longitudinal oscillations at sum and difference frequencies. But since we are interested in the low frequency oscillations only, the difference frequency oscillation has been analysed. The space variation of the generated disturbance is of the form $\exp\{-i(k_e^1 + k_e^2)z\}$ and the expression for the electric field, obtained from the coupled Maxwell and Boltzmann equations, can be written as

$$E_z^{1-2} = \epsilon_l'^{-1}\Phi' A_1^1 A_1^{2*} \tag{6}$$

where

$$\begin{aligned}\Phi' = &-\frac{i\omega_p^2 e}{18m}(k_e^1 + k_e^2)^{-2}\langle v^{-2}\rangle \\ &\times\left\{-\frac{5}{2}\left(\frac{k_e^1}{\omega_1(\omega_2-\omega_{ce})} + \frac{k_e^2}{\omega_2(\omega_1-\omega_{ce})}\right) + \frac{k_e^1 + k_e^2}{(\omega_1-\omega_{ce})(\omega_2-\omega_{ce})}\right. \\ &\left. -\frac{k_e^1}{(\omega_1-\omega_{ce})^2} - \frac{k_e^2}{(\omega_2-\omega_{ce})^2}\right\} - \frac{i\omega_p^2 e(k_e^1+k_e^2)^{-1}}{6m(\omega_1-\omega_2)} \\ &\times k_e^1 k_e^2\left(\frac{1}{\omega_1(\omega_2-\omega_{ce})^2} + \frac{1}{\omega_2(\omega_1-\omega_{ce})^2}\right) - \frac{i\omega_p^2 em}{4m_i^2(\omega_1-\omega_2)^2} \\ &\times\left(\frac{k_e^1}{\omega_1(\omega_2+\omega_{ci})} - \frac{k_e^2}{\omega_2(\omega_1+\omega_{ci})}\right)\end{aligned} \tag{7}$$

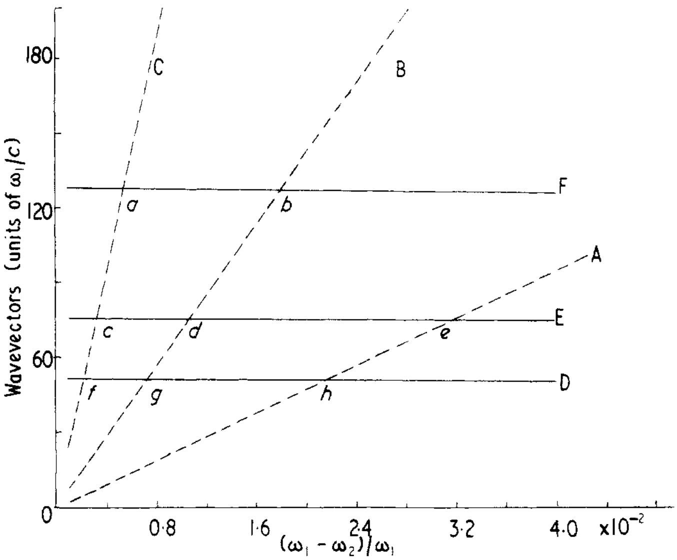

Figure 2. Wavevectors against the normalized frequency of the generated signal: curves A, B and C refer to k' in units of ω_1/c for $c/v_{th} = 10^2$, 3×10^2, 10^3 and curves D, E and F refer to $(k_e^1+k_e^2)$ (cf equation (1)) in units of ω_1/c for $\omega_p/\omega_e = 2$, $\omega_e/\omega_1 = 360$; $\omega_p/\omega_e = 2$, $\omega_p/\omega_1 = 900$; $\omega_p/\omega_e = 5$, $\omega_p/\omega_1 = 900$ respectively.

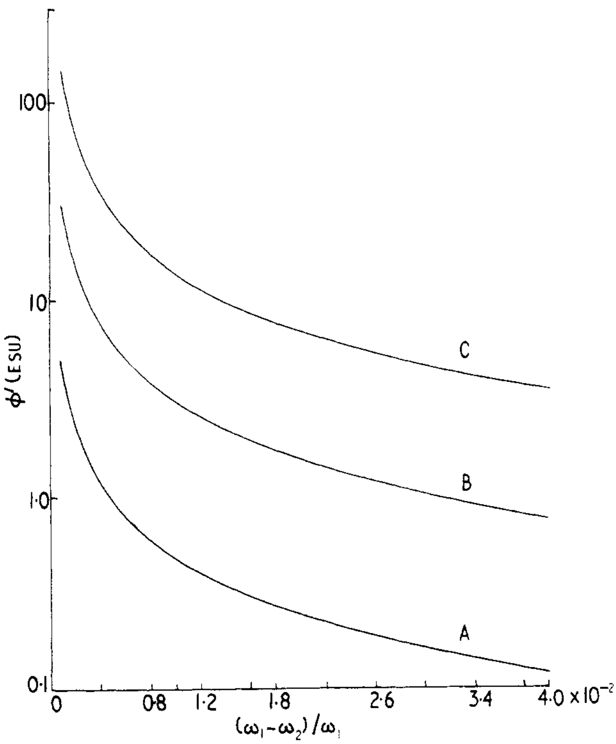

Figure 3. Φ' in units of $(e/m\omega_1)\times10^{-8}$ against $(\omega_1-\omega_2)/\omega_1$; curves A, B and C refer to $\omega_p/\omega_e = 2$, $\omega_p/\omega_1 = 360$; $\omega_p/\omega_e = 5$, $\omega_p/\omega_1 = 900$ and $\omega_p/\omega_e = 2$, $\omega_p/\omega_1 = 900$.

and ϵ_l' is the same as ϵ_l with ω replaced by $\frac{1}{2}(\omega_1-\omega_2)$ and (k_e+k_o) replaced by $(k_e^1+k_e^2)$. The plot of Φ' as a function of wave frequency is given in figure 3 when $\epsilon_l' \simeq 0{\cdot}1$. Figure 2 illustrates the possibility of resonance; the points *a*... *h* correspond to the resonance. It can be seen from these curves that in the case of the two whistler interactions, the resonance conditions are satisfied easily and the yield is also appreciably high.

5. Conclusions

In a nonisothermal dense plasma, the mutual interaction of two modes of a VLF wave gives rise to appreciably high amplitudes of the generated signal when the resonance conditions are nearly met. An alternate way to excite ion plasma oscillations is by the mutual interaction of two whistler waves. The latter method is more advantageous, since the resonant conditions are easily satisfied, even in a rare plasma, over a wide range of electron temperature, electron concentration and magnetic field strength. The amplitude of the generated signal is seen to decrease continuously with its frequency as well as the strength of the static magnetic field.

Acknowledgments

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References

Akhiezer A I, *et al* 1967 *Collective Oscillations in a Plasma* (Oxford: Pergamon Press).
Arunasalam V and Brown S C 1964 *Bull. Am. Phys. Soc.* **2** 494.
Chen Y G, Leheny R F and Marshall T C 1965 *Phys. Rev. Lett.* **15** 184–7.
Drummond W E 1962 *Phys. Fluids* **5** 1133–4.
Ginzburg V L 1960 *The Propagation of Electromagnetic Waves in Plasma* (Oxford: Pergamon Press) chap 3.
Ichimaru S, Pines D and Rostoker N 1962 *Phys. Rev. Lett.* **8** 231–3.
Kroll N M, Ron A and Rostoker N 1964 *Phys. Rev. Lett.* **13** 83–6.
Kuhn L, Leheny R F and Marshall T C 1968 *Phys. Fluids* **11** 2440–8.
Shkarofsky I P 1968 *Plasma Phys.* **10** 169–93.
Shkarofsky I P, Johnston T W and Bachynski M P 1966 *The Particle Kinetics of Plasmas* (New York: Addison Wesley).
Stix T H 1962 *The Theory of Plasma Waves* (New York: McGraw-Hill).